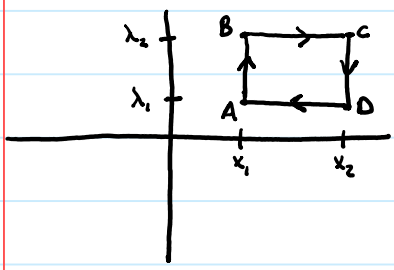
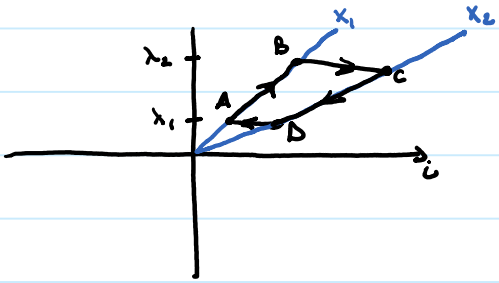
Ex] compute EFM directly for previous example



Solution: convert λ-x plane to λ-i plane

$$i = \left(\frac{1+\frac{x}{a}}{L_0}\right)\lambda$$



$$EFE|_{cycle} = \oint i\,d\lambda = \int_A^B \left(\frac{1+\frac{x}{a}}{L_0}\right)\lambda d\lambda + \int_B^C \left(\frac{1+\frac{x}{a}}{L_0}\right)\lambda d\lambda + \int_C^D \left(\frac{1+\frac{x}{a}}{L_0}\right)\lambda d\lambda + \int_D^A \left(\frac{1+\frac{x}{a}}{L_0}\right)\lambda d\lambda$$

$$= \int_{\lambda_1}^{\lambda_2} \left(\frac{1+\frac{x_1}{a}}{L_0}\right)\lambda d\lambda + \int_{\lambda_2}^{\lambda_2} \left(\frac{1+\frac{x}{a}}{L_0}\right)\lambda d\lambda + \int_{\lambda_2}^{\lambda_1} \left(\frac{1+\frac{x_2}{a}}{L_0}\right)\lambda d\lambda + \int_{\lambda_1}^{\lambda_1} \left(\frac{1+\frac{x}{a}}{L_0}\right)\lambda d\lambda$$

$$= \frac{1}{2}\left(\frac{1+\frac{x_1}{a}}{L_0}\right)\left(\lambda_2^2-\lambda_1^2\right) + \frac{1}{2}\left(\frac{1+\frac{x_2}{a}}{L_0}\right)\left(\lambda_1^2-\lambda_2^2\right) \Rightarrow \frac{1}{2}\left(\frac{1+\frac{x_1}{a}}{L_0}\right)\left(\lambda_2^2-\lambda_1^2\right) - \frac{1}{2}\left(\frac{1+\frac{x_2}{a}}{L_0}\right)\left(\lambda_2^2-\lambda_1^2\right)$$

$$EFE|_{cycle} = -\frac{(x_2-x_1)}{2L_0a}\left(\lambda_2^2-\lambda_1^2\right)$$

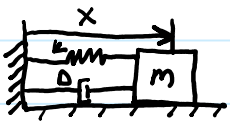
$$\boxed{EFM|_{cycle} = \frac{(x_2-x_1)}{2L_0a}\left(\lambda_2^2-\lambda_1^2\right)}$$

Lumped Mechanical Elements:

Springs : —ww— Dampers: —[]—

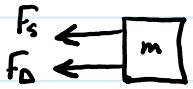
* A spring acts to keep system in equilibrium position
* Damper opposes changes in motion

Determining Forces: Ex



Spring has unstretched length l

F.B.D. :



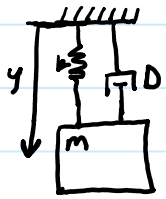
$|F_s| = k(x-l)$ * Valid for small displacements about l

$|F_D| = D\dot{x}$ * Valid for small velocity

Newton's 2nd Law: $\boxed{m\ddot{x} = -k(x-l) - D\dot{x}}$

* Can also define displacement from equilibrium: $y = x - l$ $\quad \dot{y} = \dot{x}$ $\quad \boxed{m\ddot{y} = -ky - D\dot{y}}$

Ex



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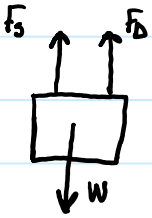
unstretched length l

F.B.D. :



Newton's 2nd Law: $m\ddot{y} = mg - k(y-l) - D\dot{y}$

Equilibrium: $\ddot{y} = \dot{y} = 0 \Rightarrow mg - k(y_{eq} - l) = 0$

$mg - ky_{eq} + kl = 0 \Rightarrow y_{eq} = l + \frac{mg}{k}$

Displacement about equilibrium: $y = z + y_{eq}$

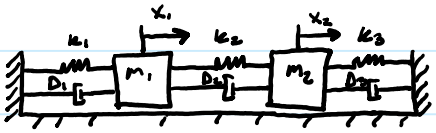
$\dot{y} = \dot{z}$

$m\ddot{z} = mg - k(z + y_{eq} - l) - D\dot{z} \Rightarrow m\ddot{z} = -kz + (mg + kl - ky_{eq}) + D\dot{z}$

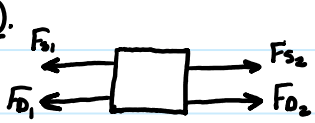
$\boxed{m\ddot{z} = -kz + D\dot{z}}$

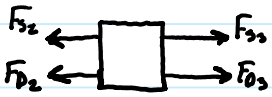
Multiple degrees of freedom

Ex



F.B.D.





$|F_{s_1}| = k_1(x_1 - 0)$ $\quad |F_{s_2}| = k_2(x_2 - x_1)$ $\quad |F_{s_3}| = k_3(0 - x_2)$

$|F_{D_1}| = D_1(\dot{x}_1 - 0)$ $\quad |F_{D_2}| = D_2(\dot{x}_2 - \dot{x}_1)$ $\quad |F_{D_3}| = D_3(0 - \dot{x}_2)$

$$m_1\ddot{x}_1 = k_2(x_2 - x_1) - k_1(x_1 - 0) + D_2(\dot{x}_2 - \dot{x}_1) - D_1(\dot{x}_1 - 0)$$

$$m_2\ddot{x}_2 = k_3(0 - x_2) - k_2(x_2 - x_1) + D_3(0 - \dot{x}_2) - D_2(\dot{x}_2 - \dot{x}_1)$$

$$m_1\ddot{x}_1 = -(k_1 + k_2)x_1 - (D_1 + D_2)\dot{x}_1 + k_2 x_2 + D_2\dot{x}_2$$

$$m_2\ddot{x}_2 = -(k_2 + k_3)x_2 - (D_2 + D_3)\dot{x}_2 + k_2 x_1 + D_2\dot{x}_1$$

Matrix Form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} = \begin{bmatrix} -(k_1+k_2) & k_2 \\ k_2 & -(k_2+k_3) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} -(D_1+D_2) & D_2 \\ D_2 & -(D_2+D_3) \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$$

$$\underline{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \qquad \underline{\underline{M}} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$\underline{\underline{K}} = \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & (k_2+k_3) \end{bmatrix} \qquad \underline{\underline{D}} = \begin{bmatrix} (D_1+D_2) & -D_2 \\ -D_2 & (D_2+D_3) \end{bmatrix}$$

$$\boxed{\underline{\underline{M}}\,\ddot{\underline{x}} = -\underline{\underline{K}}\,\underline{x} - \underline{\underline{D}}\,\dot{\underline{x}}}$$